Problem Assignments for Homework No. 1
Due by 1:00pm on Friday January 26 2018

Homework Policy

Solutions to the written homeworks should be submitted to the drop-off box located on the 1st floor of Etcheverry Hall by 1:00pm. The homework will be picked up by 1:00pm and late homeworks will *not* be graded. You are also expected to complete the electronic homework problems by 1:0pm on the same day that homework is due. We expect to return the corrected homeworks to you promptly and the solutions will be posted on **bcourses** by 3:00pm on the days the homework is due.

The E-problems can be accessed from your account at

WileyPlus

Here are some key points to take into consideration when submitting your homework solutions:

1. While I expect you to work with other students in the class on the homework, the final submitted solution should be yours alone. If we find violations of this policy, then the homeworks in question will be returned ungraded, and the possibility of not grading the student(s) homework for the remainder of the semester will be considered.
2. Where appropriate, your MATLAB/MATHEMATICA code should be submitted with your homework solutions. The code should be commented and contain your name and SID.
3. Points total for each homework problem are given below. Most of the points will be assigned for the method you use to solve the problem and not merely on getting the correct numerical answer.
4. No credit will be given for untidy and/or illegible homework solutions.
5. All vectors should be underlined.

Class Announcements

The discussion sessions will start during the second week of the semester. Any changes in times for these sessions, and additional sessions, will be announced on the ME104 website:

https://bcourses.berkeley.edu/

Homework 1

8 Questions

(40 + 100 Points)

1. Problem 2/9 *(E-problem 10 Points)*

For this problem, choose the origin so that $\mathbf{r} = x\mathbf{E}_x$. You are given $\mathbf{r}(0)$, $\mathbf{v}(0)$, $\mathbf{r}(T)$, $\mathbf{v}(T)$ and asked to calculate $\mathbf{a}$.

2. Problem 2/14 *(E-problem 10 Points)*

For this problem, choose the axes and origin so that $\mathbf{r} = y\mathbf{E}_y$ with $\mathbf{E}_y$ pointing upwards. Note that you are given $\mathbf{r}(0)$, $\mathbf{v}(0)$ and $\mathbf{a}$ and asked to solve for the maximum height reached and time of flight.

3. Problem 2/31 *(E-problem 10 Points)*

This is a problem of interest to those designing the hyperloop. Note that the limiting acceleration is not very large. If you have time, you might find it interesting to look at the related problem 2/56.

4. Problem 2/35 *(E-problem 10 Points)*

For this problem, you are given the path, the initial velocity, and the acceleration as a function of velocity. Note the similarity of this problem to 2/49.

5. Problem 2/36 *(10 Points)* Discussion Session Problem

Here, you are told that $\mathbf{v}(0) = v_0\mathbf{E}_x$. You can pick the origin so that $\mathbf{r}(t) = x\mathbf{E}_x$ with $x = 0$ at the instant of impact.

Note that the acceleration is a function of v. Hence using the identity

$$a = \frac{dv}{dx}v \tag{1}$$

it is easy to integrate and find $v(x)$. The answer should then be within easy reach.

If you want to get more out of this problem, form a matrix-vector equation $\mathsf{Ab} = \mathsf{c}$ from the two equations that help you to solve k_1 and k_2, by inverting the matrix you can then solve for the unknown $\mathsf{b} = [k_1, k_2]^T$. You should look at the related problems 2/39 and 2/40.

6. Problem 2/88 (MATLAB Required) *(30 Points)* Discussion Session Problem

To get *any* credit for the solution you submit to this problem you should use the 4 steps **and** complete the Matlab assignment. No partial credit will be given.

On your way to the answer, you will find that

$$\mathbf{r}(t) = v_0 \cos(30^\circ)\, t\mathbf{E}_x + \left(v_0 \sin(30^\circ)\, t - \frac{g}{2}t^2\right)\mathbf{E}_y. \tag{2}$$

Letting the times of flight be T, you will find that there are two cases to consider. For both of these cases, you will find that the range $R = x(T)$ is

$$R = \cot(30^\circ)\left(h + \frac{g}{2}T^2\right) \quad \text{(meters)}, \tag{3}$$

where $h = -0.1$ meters and $g = 9.81$ meters per second per second. The answer to the problem is

$$6.15 \le v_0 \le 6.68 \quad \text{(meters per second)}. \tag{4}$$

You should notice that the time of flight T increases as v_0 increases.

Matlab: Show that the solution (4) is correct by providing graphs of

$$x(t),\ y(t) \text{ and } y(x), \text{ for } t \in [0, T].$$

For these graphs, you should pick 5 representative values for v_0. For three of these values the ball should reach the box and for the remaining values of v_0 the ball should fail to reach the box. Your three graphs ($x(t)$, $y(t)$ and $y(x)$) should all be printed on the same page, the axes must be labeled, the location of the box explicitly shown, and the values of v_0 and T noted.

To get more out of this problem, notice the similarities to 2/84, 2/87, 2/90, and 2/93, among many others in this chapter of the textbook.

7. Problem 2/95 (Matlab Needed) *(30 Points)*

To get *any* credit for the solution you submit to this problem you should use the 4 steps and complete the Matlab assignment. No partial credit will be given.

It's convenient to choose $\mathbf{E}_x$ to point up the incline. With this choice, on your way to the answer, you will find the differential equations

$$\ddot{x} = -g\sin(\alpha), \qquad \ddot{y} = -g\cos(\alpha). \tag{5}$$

You can now solve for the range R as a function of θ given $\mathbf{v}(0)$ - be careful with the angle $\theta - \alpha$. For a given value of α you then need to compute the value of θ which maximizes R. We denote the resulting value of θ and the maximum value of R by θ^* and R^*, respectively. Your expression for R^* should be surprisingly simple (and not have any singularities when $\alpha = \frac{\pi}{2}$), and as a check on your answer for $\alpha = 0$, the maximum range occurs when $\theta^* = 45°$.

Matlab: For a range of values of α between $0°$ and $90°$ provide a plot of the maximum range R^*. You will find it convenient to plot the dimensionless quantity. $\frac{R^* g}{v_0^2}$. You should verify that the solution is continuous at $\alpha = \frac{\pi}{2}$. For full credit, give an interpretation of the resulting plot: in particular give an interpretation of your results when $\alpha = \frac{\pi}{2}$ and plot $\mathbf{r}(t)$ for various values of α.

8. Problem 2/96 (Matlab Required) *(30 Points)*

To get *any* credit for the solution you submit to this problem you should use the 4 steps and complete the Matlab assignment. No partial credit will be given.

On your way to the answer, you will find the differential equations

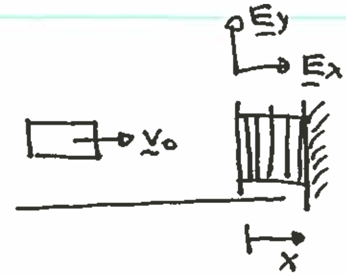
$$\ddot{x} = -k\dot{x}, \qquad \ddot{y} = -k\dot{y} - g. \tag{6}$$

It is expeditious to solve for $v_y(t) = \dot{y}(t)$ first. Another integration then gives $y(t)$. The solutions for these two variables can then easily be transformed to those for $\dot{x}(t)$ and $x(t)$.

Matlab: Provide graphs of $x(t)$, $y(t)$ and $y(x)$. For these graphs, you should pick representative values for v_0 and θ, and by increasing k from 0 in 5 increments show how the drag influences the path of the projectile. Your three graphs should all be printed on the same page, the axes should be labeled and the values of v_0, θ, and k that you chose should be noted.

Show that the solution for $k = 0$ cannot be found as a limiting case from the solution for the case $k \neq 0$.

Problem 2/36
M, K J B : 8th Edition



Given $\quad \underline{v}_0 = 3.25\,\underline{E}_x$

$$\underline{a} = -(K_1 x + K_2 x^3)\underline{E}_x$$

$$\underline{v}(x = x_{max}) = \underline{0} \qquad x_{max} = 0.475\text{ m}$$

$$\underline{v}(x = 0.5x_{max}) = 2.85\,\underline{E}_x$$

Determine $\quad K_1$ and K_2

Solution $\quad \ddot{\underline{r}} = a\,\underline{E}_x = -(K_1 x + K_2 x^3)\underline{E}_x$

Using identity $\quad a = v\dfrac{dv}{dx}$ and initial condition we find that

$$v^2 - v_0^2 = -K_1 x^2 - \frac{K_2}{2}x^4$$

When deflection is maximum, $v = 0$. Using $v = 2.85$ when $x = \dfrac{x_{max}}{2}$

then gives 2 equations for the 2 unknowns K_1 and K_2:

$$\begin{bmatrix} x_{max}^2 & +\frac{1}{2}x_{max}^4 \\ \frac{x_{max}^2}{4} & \frac{x_{max}^4}{32} \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} v_0^2 \\ v_0^2 - (2.85)^2 \end{bmatrix}$$

Inverting matrix and inserting numbers gives.

$$K_1 = 42.072\text{ N/m kg}$$

$$K_2 = 42.0377\text{ N/m}^3\text{ kg}$$

Problem 2.188
M, K & B 3th Edition

Given $\underline{v}_0 = v_0 \cos 30^\circ \underline{E}_x + v_0 \sin 30^\circ \underline{E}_y$

$\theta_0 = 30^\circ$

Determine Range of speeds v_0 such that

$\underline{r}(T)\cdot\underline{E}_y = h$ when $\underline{x}(T)\cdot\underline{E}_y \in [3.5, 3.5+w]$

$w = .6$ meters

$h = -0.1$ meters

Solution

I Choose origin O such that $\underline{r}(t=0) = \underline{0}$

choose $(\underline{E}_x, \underline{E}_y, \underline{E}_z)$ such that $\underline{v}(t=0) = v_0 \cos\theta^\circ \underline{E}_x + v_0 \sin\theta^\circ \underline{E}_y$

$$\underline{r}(t) = x\underline{E}_x + y\underline{E}_y \Rightarrow \underline{a}(t) = \ddot{x}\underline{E}_x + \ddot{y}\underline{E}_y$$

II $-mg\underline{E}_y$

III $\underline{F} = m\underline{a}$:

$$m\ddot{y} = -mg$$
$$m\ddot{x} = 0$$
$$m\ddot{z} = 0$$

IV Solving for $\underline{r}(t)$

$$x = V_{0x} t + x(0)$$
$$= V_0 \cos\theta^\circ \, t$$
$$y = -\frac{gt^2}{2} + V_{0y} t + y(0)$$
$$= V_0 \sin\theta^\circ t - gt^2/2$$

Imposing boundary condition $x(0) = 0,\ y(0) = 0,$

$x(T) = L + u \qquad u \in [0, w]$

$y(T) = -h$

$$\left.\begin{aligned} h &= \frac{gT^2}{2} - V_0 \sin\theta^\circ T \\ L + u &= V_0 \cos\theta^\circ T \end{aligned}\right] \quad \text{2 unknowns } V_0, T$$

Solving for V_0:

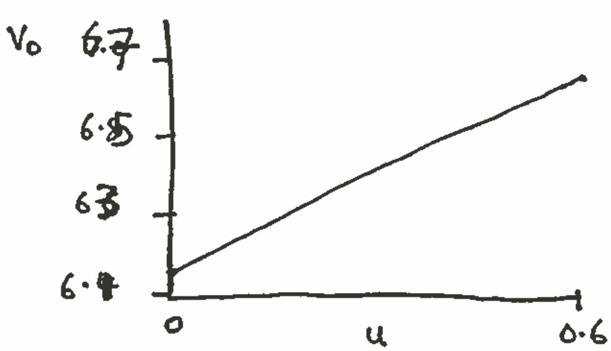
$$h = \frac{g}{2}\left(\frac{L+u}{V_0 \cos\theta^\circ}\right)^2 - V_0 \sin\theta^\circ \left(\frac{L+u}{V_0 \cos\theta^\circ}\right)$$

Hence

$$h + (L+u)\tan\theta^\circ = \frac{g}{2}\left(\frac{L+u}{V_0 \cos\theta^\circ}\right)^2$$

So

$$V_0 = \left(\sqrt{\left(h + (L+u)\tan\theta^\circ\right) \frac{2\cos^2\theta^\circ}{g(L+u)^2}}\right)^{-1}$$

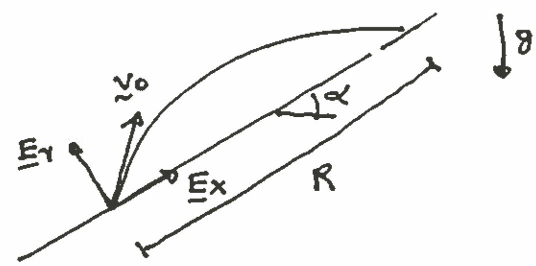


Solving for V_0:

$V_0(u=0) = 6.45$ m/sec

$V_0(u = 600\text{mm}) = 6.68$ m/sec

Problem 2/95



Given $\quad \underset{\sim}{v_0} = v_0 \cos(\theta - \alpha) \underline{E}_x + v_0 \sin(\theta-\alpha) \underline{E}_Y$

Determine $\quad R_{max}$ for given value of α by varying θ.

Evaluate answers for $\alpha = 0, 30°, 45°$

Solution

I $\quad$ Choose 0 such that $\underline{r}(0) = \underline{0}$

Note that $\underline{v}(0) = v_0 \cos(\theta-\alpha)\underline{E}_x + v_0 \sin(\theta-\alpha)\underline{E}_Y$

$\underline{r}(t) = x\underline{E}_x + y\underline{E}_Y \qquad \underline{a}(t) = \ddot{x}\underline{E}_x + \ddot{y}\underline{E}_Y$

II $\quad -mg\cos\alpha\, \underline{E}_Y - mg\sin\alpha\, \underline{E}_x$

III $\quad \underline{F} = m\underline{a}$

$\cdot \underline{E}_x \qquad m\ddot{x} = -mg\sin\alpha$

$\cdot \underline{E}_Y \qquad m\ddot{y} = -mg\cos\alpha$

IV $\quad$ From $\underline{F} = m\underline{a}$ $\qquad \ddot{x} = -g\sin\alpha$
$\qquad \ddot{y} = -g\cos\alpha$

Hence using $\underline{r}(0) = \underline{0}$

$$x(t) = -g\sin\alpha\frac{t^2}{2} + v_0 \cos(\theta-\alpha)t$$
$$y(t) = -g\cos\alpha\frac{t^2}{2} + v_0 \sin(\theta-\alpha)t$$

Hence range R and time of flight T are determined from

$$X(T) = R \quad \text{and} \quad y(T) = 0$$

$$\Rightarrow \qquad R = -g \sin\alpha \frac{T^2}{2} + V_0 \cos(\theta - \alpha)\, T$$

$$0 = -g \cos\alpha \frac{T^2}{2} + V_0 \sin(\theta - \alpha)\, T$$

This gives 2 equations for 2 unknowns. Ignoring the case $R = 0$ and $T = 0$

$$T = \frac{2 V_0 \sin(\theta - \alpha)}{g \cos\alpha}$$

$$R = \frac{2 V_0^2}{g} \cos\theta \sec^2\alpha \sin(\theta - \alpha) \qquad (*)$$

To find max R for given α we solve $\frac{\partial R}{\partial \theta} = 0$ for $\theta = \theta^*$

$$\frac{\partial R}{\partial \theta} = \frac{2 V_0^2}{g} \cos(\alpha - 2\theta) \sec^2\alpha \quad \Rightarrow \quad \frac{\partial R}{\partial \theta} = 0 \text{ when } \cos(\alpha - 2\theta^*) = 0$$

and $\sec\alpha \neq \infty$

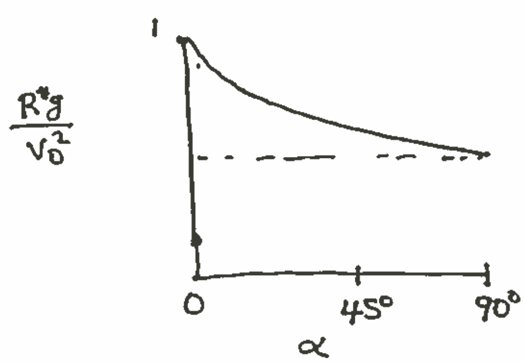
Hence $\quad \theta^* = \mp \frac{\pi}{4} + \frac{\alpha}{2}$

However $\alpha \in (0, \pi/2)$ and we need T and R to be positive so

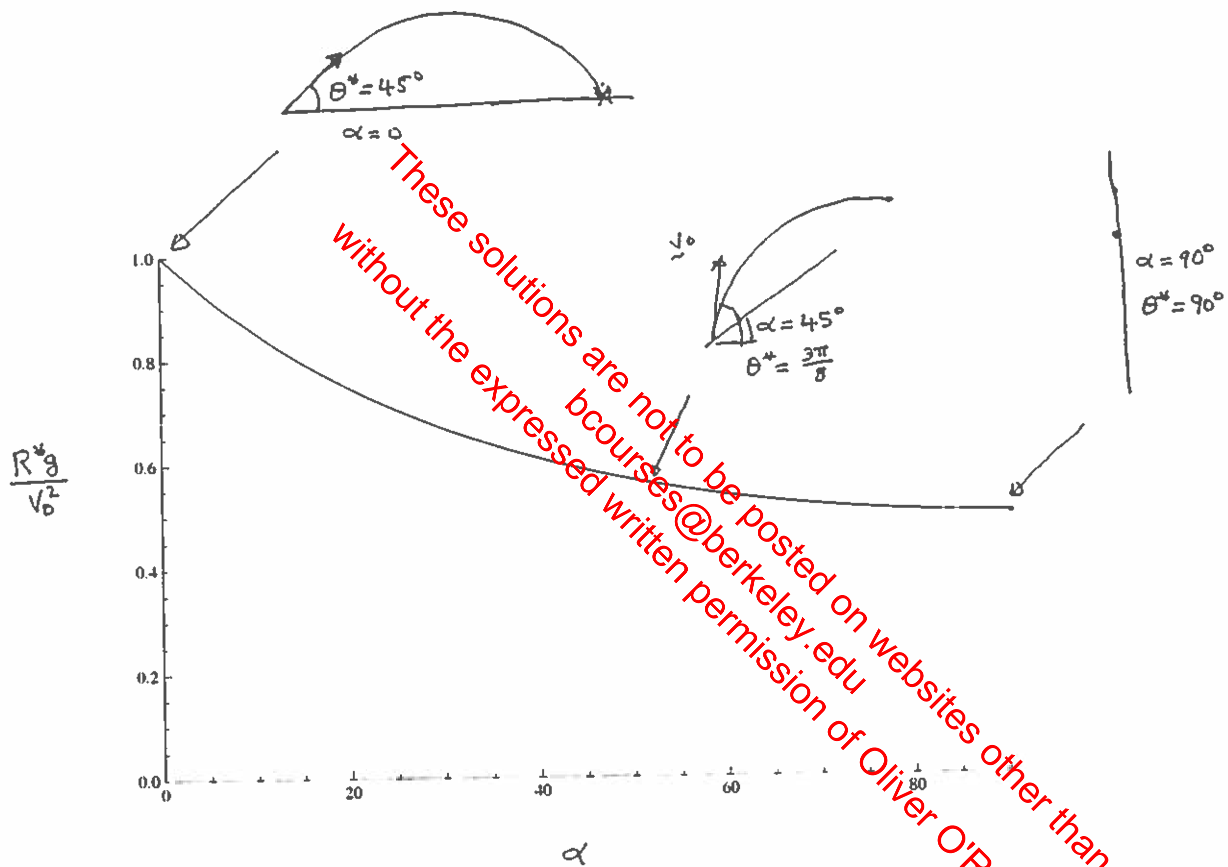
$$\theta^* = \pi/4 + \frac{\alpha}{2}$$

maximum range R^* is found by substituting θ^* for θ in $(*)$

$$R^* = \frac{V_0^2}{g} \frac{1}{1 + \sin\alpha}$$



See next page for details on plot.



$$R^* = \frac{V_0^2}{g} \frac{1}{1 + \sin\alpha}$$

$$\theta^* = \frac{\alpha}{2} + \frac{\pi}{4}$$

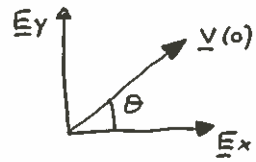
For $\alpha = 0$, we get the classic result that the range is maximized when $\theta^* = \pi/4$

For $\alpha = \pi/2$, the range R is the height that the particle moves up before descending.

Problem 2196

Given: $\underline{v}_o = v_o(\cos\theta\,\underline{E}_x + \sin\theta\,\underline{E}_Y)$

$\underline{a} = -K\underline{v} - g\,\underline{E}_Y$



Determine: $\underline{v}(t)$

$\underline{v}$ terminal

Solution: I: Let $\underline{r} = x\underline{E}_x + y\underline{E}_Y$, $\underline{r}(0) = \underline{0}$, $\dot{\underline{r}}(0) = \underline{v}(0) = \underline{v}_o$

We omit steps II, III as they are similar to previous 2 problems

IV: Hence $\ddot{x} = -K\dot{x}$, $\ddot{y} = -(K\dot{y} + g)$

Now $\dot{y} = v_y \Rightarrow \dot{v}_y = -(Kv_y + g) \Rightarrow -\int_{v_{oy}}^{\dot{y}} \frac{dv_y}{Kv_y + g} = \int_0^t d\tau$

$\Rightarrow \dot{y} = \left[v_{oy} + g/K\right]e^{-Kt} - g/K$

and $y = \frac{-1}{K}\left[v_{oy} + g/K\right]e^{-Kt} - \frac{g}{K}t + C$ where C is a constant.

Now when $t=0$, $y=0$, Hence $C = \frac{1}{K}\left[v_{oy} + g/K\right]$

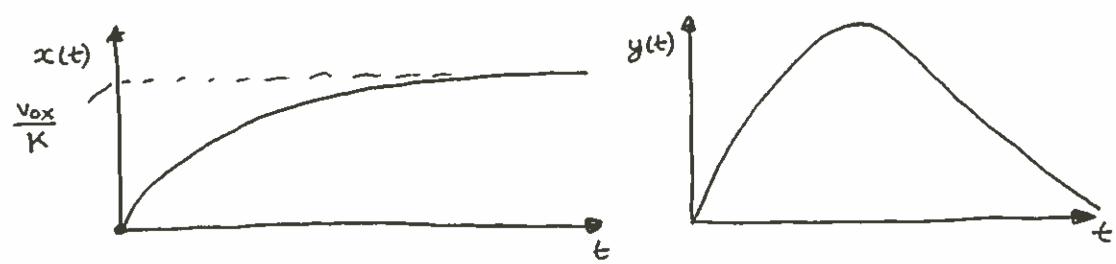
Hence $$\boxed{y(t) = \frac{1}{K}\left[v_{oy} + \frac{g}{K}\right]\left[1 - e^{-Kt}\right] - \frac{g}{K}t} \quad (*)$$

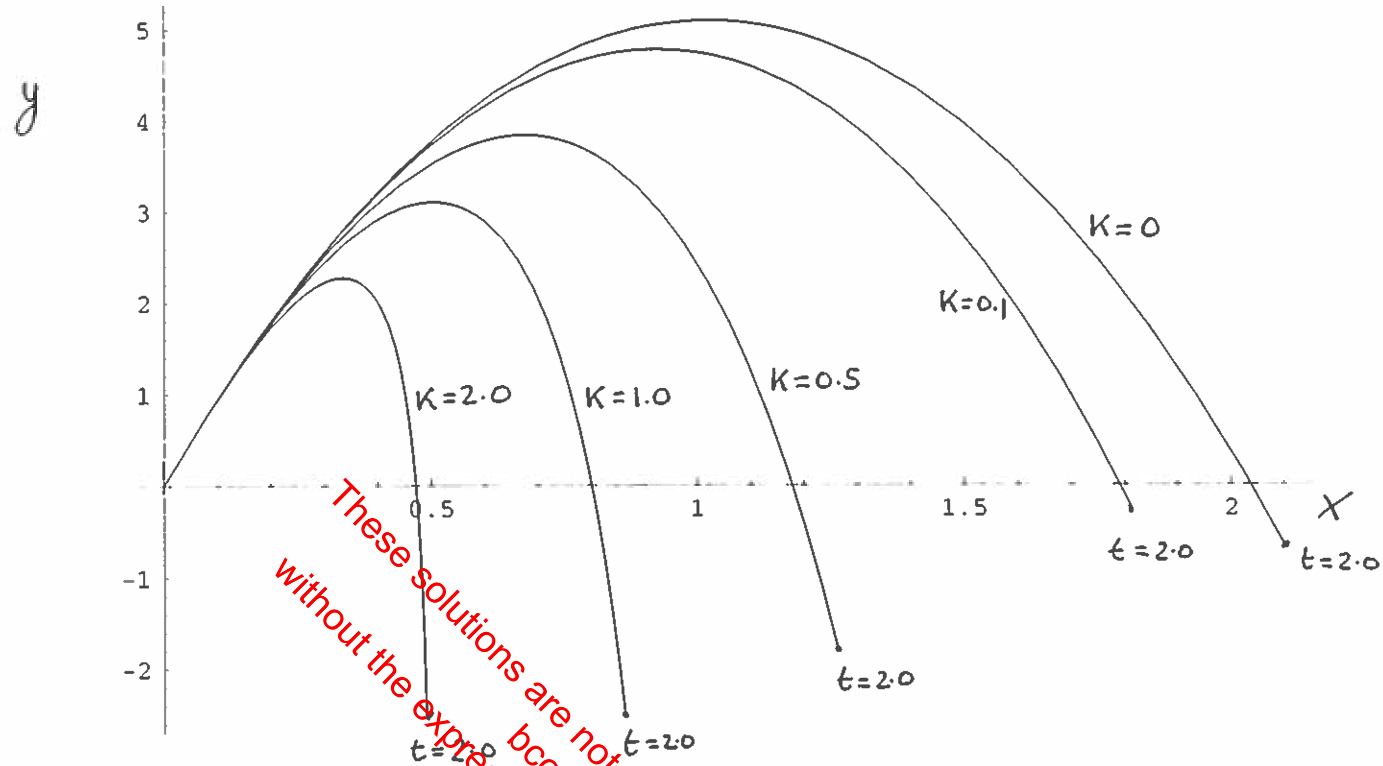
To determine the solution for $x(t)$ we set $g = 0$ in $(*)$ and $v_{oy} \to v_{ox}$

$$x(t) = \frac{1}{K}v_{ox}\left(1 - e^{-Kt}\right)$$

Terminal Velocity occurs when $t \to \infty$: $\dot{y}(t\to\infty) = -g/K$

$\dot{x}(t\to\infty) = 0.$





For all these trajectories : $g = 9.81\ m/sec^2$

$V_{0x} = 1.0\ m/sec$

$V_{0y} = 10.0\ m/sec$